

We compute now the bending moment in the beam using formula (6.18):

$$\Pi = [D][B]\{u\} = EI [B]\{u\}$$

$$\text{where (6.16) } [B] = \begin{bmatrix} \left(-\frac{6}{l^2} + \frac{12x}{l^3}\right) & \left(-\frac{4}{l} + \frac{6x}{l^2}\right) & \left(\frac{6}{l^2} - \frac{12x}{l^3}\right) & \left(-\frac{2}{l} + \frac{6x}{l^2}\right) \end{bmatrix}$$

element 1:

$$\Pi(x) = EI [B] \cdot \begin{bmatrix} v_1 \\ \psi_1 \\ v_2 \\ \psi_2 \end{bmatrix} = [B] \begin{bmatrix} -\frac{ql^4}{10EI} - \frac{\pi l^2}{10EI} \\ 0 \\ 0 \\ \frac{ql^3}{5EI} + \frac{\pi l}{5EI} \end{bmatrix} \cdot EI$$

$$= \left(-\frac{6}{l^2} + \frac{12x}{l^3}\right) \left(-\frac{ql^4}{10EI} - \frac{\pi l^2}{10EI}\right) + \left(-\frac{2}{l} + \frac{6x}{l^2}\right) \left(\frac{ql^3}{5EI} + \frac{\pi l}{5EI}\right) EI$$

$$= -\frac{6}{l^2} \dots = 0.2 ql^2 + 0.2 \pi \text{ constant}$$

element 2:

$$\Pi(x) = EI [B] \begin{bmatrix} v_2 \\ \psi_2 \\ v_3 \\ \psi_3 \end{bmatrix} = [B] \begin{bmatrix} 0 \\ \frac{ql^3}{5EI} + \frac{\pi l}{5EI} \\ \frac{19ql^4}{120EI} + \frac{\pi l^2}{5EI} \\ \frac{ql^3}{60EI} + \frac{\pi l}{10EI} \end{bmatrix} \cdot EI$$

$$= \left[\left(-\frac{4}{l} + \frac{6x}{l^2}\right) \left(\frac{ql^3}{5EI} + \frac{\pi l}{5EI}\right) + \left(\frac{6}{l^2} - \frac{12x}{l^3}\right) \left(\frac{19ql^4}{120EI} + \frac{\pi l^2}{5EI}\right) + \left(-\frac{2}{l} + \frac{6x}{l^2}\right) \left(\frac{ql^3}{60EI} + \frac{\pi l}{10EI}\right) \right] EI$$

$$= \left(-\frac{4}{l} \cdot \frac{l}{5} + \frac{6}{l^2} \cdot \frac{19l^2}{120} - \frac{2}{l} \cdot \frac{l}{60}\right) ql^2 + \left(\frac{6x}{l^2} \cdot \frac{l}{5} - \frac{12x}{l^3} \cdot \frac{19l^2}{120} + \frac{6x}{l^2} \cdot \frac{l}{60}\right) ql^2 +$$

$$\dots = \left(\frac{7}{60} - \frac{3}{5} \frac{x}{l}\right) ql^2 + \left(\frac{1}{5} - \frac{3}{5} \frac{x}{l}\right) \pi$$