

In the element:

$$u = [N_1 \quad N_2 \quad N_3] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = [B] \{u\}$$

$$[B] = \begin{bmatrix} \frac{4x}{l^2} - \frac{1}{l} & -\frac{8x}{l^2} & \frac{4x}{l^2} + \frac{1}{l} \end{bmatrix}$$

The matrix  $[D]$  is the scalar Young's modulus.

Therefore

$$\int_{-l/2}^{l/2} [B]^T [D] [B] A dx = \int_{-l/2}^{l/2} \begin{bmatrix} \frac{4x}{l^2} - \frac{1}{l} \\ -\frac{8x}{l^2} \\ \frac{4x}{l^2} + \frac{1}{l} \end{bmatrix} EA \begin{bmatrix} \frac{4x}{l^2} - \frac{1}{l} & -\frac{8x}{l^2} & \frac{4x}{l^2} + \frac{1}{l} \end{bmatrix} dx$$

$$= EA \int_{-l/2}^{l/2} \begin{bmatrix} \left(\frac{4x}{l^2} - \frac{1}{l}\right)^2 & -\frac{8x}{l^2} \left(\frac{4x}{l^2} - \frac{1}{l}\right) & \left(\frac{4x}{l^2} - \frac{1}{l}\right) \left(\frac{4x}{l^2} + \frac{1}{l}\right) \\ \left(\frac{8x}{l^2}\right)^2 & -\frac{8x}{l^2} \left(\frac{4x}{l^2} + \frac{1}{l}\right) & \left(\frac{4x}{l^2} + \frac{1}{l}\right)^2 \end{bmatrix} dx$$

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